THE BOREL CONJECTURE AND ASPHERICAL 4-MANIFOLDS

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A space is aspherical if its universal cover is contractible. Obstruction theory shows that any two aspherical CW complexes with isomorphic fundamental groups are homotopy equivalent. The Borel Conjecture states that any two closed aspherical manifolds with isomorphic fundamental groups are homeomorphic. The Borel Conjecture for manifolds with boundary states that two aspherical compact manifolds with isomorphic fundamental groups and homeomorphic boundaries are in fact homeomorphic.

Topology surgery shows that the Borel Conjecture is valid in dimension 4 for manifolds with good (essentially elementary amenable) fundamental groups. Work in progress with Jonathan Hillman examines the possible fundamental groups and the possible homeomorphism classes of the boundary of compact aspherical 4-manifolds.