NEW RESULTS ON ONE AND TWO FIXED POINT ACTIONS ON SPHERES

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It is known that a finite group G admits a smooth one fixed point action on a sphere if and only if G is an Oliver group. One can ask therefore a more specific question: what are the dimensions of spheres on which a given Oliver group can act smoothly with exactly one fixed point? We provide a list of specific Oliver groups and dimensions of spheres for which it was possible to exclude such actions. This part of the talk if based on the joint work with A. Borowiecka, as well as its further generalizations contained in my PhD thesis.

In the second part of the talk we focus on the Laitinen Conjecture which predicts negative answers to the Smith Question (Smith asked whether, for a finite group G acting smoothly on a sphere S with exactly two fixed points x and y, the tangent spaces T_xS and T_yS have to be isomorphic as $\mathbb{R}G$-modules). We define an infinite family of finite groups for which the Laitinen Conjecture remained unsettled and show that this conjecture holds for this family of groups. Moreover, in the case of the groups from the family, the Laitinen Conjecture implies the existence of their actions on homotopy spheres with exactly two fixed points and non-isomorphic group module structures of the tangent spaces at the two fixed points.