EQUIVARIANT SURGERY AND DIMENSION CONDITIONS

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Many researchers (e.g. J. Milnor, M. Kervaire, S. Novikov, W. Browder, C. Wall, W.-C. Hsiang, W. Sharpe, Cappell-Shaneson, D. Anderson, A. Bak, F. Connolly, S. Ferry, I. Hambleton, I. Madsen, E. Pedersen, C. Thomas, and etc.) had contributed to development of the ordinary surgery theory on compact smooth manifolds X of dimension ≥ 5 and to its applications. We can perform surgeries on the manifold X up to the middle dimension when a degree-one map $f : X \to Y$ and a bundle isomorphism $b : T(X) \oplus \varepsilon_X(\mathbb{R}^s) \to f^*\xi \oplus \varepsilon_X(\mathbb{R}^s)$, where $\xi \downarrow Y$, are given.

Let G be a finite group and X a compact connected smooth G-manifold with a G-map $f : X \to Y$ and a G-bundle isomorphism $b : T(X) \oplus \varepsilon_X(\mathbb{R}^s) \to f^*\xi \oplus \varepsilon_X(\mathbb{R}^s)$. To perform G-surgeries on X of isotropy type (H), some dimension conditions on the fixed point sets by subgroups of G must be invoked. First we require the hypothesis $\dim X_\alpha^H \geq 5$ on the relevant connected component X_α^H of X^H. T. Petrie's theory of G-surgery of isotropy type (H) works well if we invoke the strong gap condition:

$$\text{(SGC)} \quad \begin{array}{l} (1)\ 2\dim X_\beta^K + 2 < \dim X_\alpha^H \quad (H < \forall\, K \leq N_G(H),\ \forall\, X_\beta^K \subset X_\alpha^K), \text{ and} \\ (2)\ 2\dim X_\beta^K < \dim X_\alpha^H \quad (H < \forall\, K \leq G \text{ with } K \not\leq N_G(H),\ \forall\, X_\beta^K \subset X_\alpha^K). \end{array}$$

Petrie–K. Dovermann studied the induction–restriction theory of the G-surgery obstructions. We found the method killing the G-surgery obstructions by G-connected sums associated with certain idempotents of the Burnside ring of G. Morimoto–K. Pawałowski applied the method to obtain a theorem to delete (or insert) closed manifolds as connected components of G-fixed point sets from (or into) spheres for gap Oliver groups G. The result presented a passage to finding non-isomorphic Smith equivalent representations.

Generalizing Wall's surgery obstruction group by the notion 'form parameter of quadratic forms' appearing in Bak's book, we improved G-surgery theory under the gap condition:

$$\text{(GC)} \qquad 2\dim X_\beta^K < \dim X_\alpha^H \quad (H < \forall\, K \leq G,\ \forall\, X_\beta^K \subset X_\alpha^K).$$

We spent a decade to improve it to G-surgery theory under the weak gap condition:

$$\text{(WGC)} \quad \begin{array}{l} (1)\ 2\dim X_\beta^K \leq \dim X_\alpha^H \quad (H < \forall\, K \leq N_G(H),\ \forall\, X_\beta^K \subset X_\alpha^K), \text{ and} \\ (2)\ 2\dim X_\beta^K < \dim X_\alpha^H \quad (H < \forall\, K \leq G \text{ with } K \not\leq N_G(H),\ \forall\, X_\beta^K \subset X_\alpha^K). \end{array}$$

To obtain the theory, we need the notion 'positioning data' and the notion 'generalized form module with positioning data'. We should remark that Dovermann had already studied a C_2-surgery theory under the weak gap condition, where C_2 is the group of order 2.

In this talk, we revisit the G-surgery theories and review several applications.