# THE LOOP SPACE HOMOLOGY OF A SMALL CATEGORY

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In an article published in 2009, Dave Benson described, for a finite group $G$, the mod $p$ homology of the space $\Omega(BG_p^\wedge)$ — the loop space of the $p$-completion of $BG$ — in purely algebraic terms. In joint work with Carles Broto and Ran Levi, we have tried to better understand Benson's result by generalizing it. Among other things, we showed that when $\mathcal{C}$ is a small category, $|\mathcal{C}|$ is its geometric realization, $R$ is a commutative ring, and $|\mathcal{C}|_R^+$ is a plus construction of $|\mathcal{C}|$ with respect to homology with coefficients in $R$, then $H_*(\Omega(|\mathcal{C}|_R^+); R)$ is the homology any chain complex of projective $R\mathcal{C}$-modules that satisfies certain conditions. Benson's theorem is then the special case where $\mathcal{C}$ is the category associated to a finite group $G$ and $R = \mathbb{F}_p$, and thus $p$-completion appears as a special case of the plus construction.