HOLOMORPHIC FOLIATIONS ON COMPLEX MOMENT-ANGLE MANIFOLDS

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Moment-angle manifolds provide a wide class of examples of non-Kaehler compact complex manifolds. A complex moment-angle manifold Z is constructed via a certain combinatorial data, called a complete simplicial fan. In the case of rational fans, the manifold Z is the total space of a holomorphic bundle over a toric variety with fibres compact complex tori. In this case, the invariants of the complex structure of Z, such Dolbeault cohomology and the Hodge numbers, can be analysed using the Borel spectral sequence of the holomorphic bundle.

In general, a complex moment-angle manifold Z is equipped with a canonical holomorphic foliation $\mathcal{F}$ which is equivariant with respect to the algebraic torus action. Examples of moment-angle manifolds include the Hopf manifolds, Calabi-Eckmann manifolds, and their deformations. The holomorphic foliated manifold $(Z, \mathcal{F})$ has been also studied as a model for non-commutative toric varieties in the works several authors (arXiv:1308.2774, arXiv:1705.11110).

We construct transversely Kaehler metrics on moment-angle manifolds Z, under some restriction on the combinatorial data. We prove that all Kaehler submanifolds in such a moment-angle manifold lie in a compact complex torus contained in a fibre of the foliation $\mathcal{F}$. For a generic moment-angle manifold Z in its combinatorial class, we prove that all its subvarieties are moment-angle manifolds of smaller dimension. This implies, in particular, that Z does not have non-constant meromorphic functions, i. e. its algebraic dimension is zero.

Battaglia and Zaffran (arXiv:1108.1637) computed the basic Betti numbers for the canonical holomorphic foliation on a moment-angle manifold Z corresponding to a shellable fan. They conjectured that the basic cohomology ring in the case of any complete simplicial fan has a description similar to the cohomology ring of a complete simplicial toric variety due to Danilov and Jurkiewicz. We prove the conjecture.

The research was carried out at the Steklov Institute of Mathematics and supported by the Russian Science Foundation grant no. 20-11-19998.

The proof uses an Eilenberg–Moore spectral sequence argument; the key ingredient is the formality of the Cartan model for the torus action on Z.

The talk is based on joint works with Hiroaki Ishida, Roman Krutowski, Yuri Ustinovsky and Misha Verbitsky.

References

[1] Taras Panov, Yuri Ustinovsky and Misha Verbitsky. *Complex geometry of moment-angle manifolds.* Math. Zeitschrift 284 (2016), no. 1, 309–333.

[2] Hiroaki Ishida, Roman Krutowski and Taras Panov. *Basic cohomology of canonical holomorphic foliations on complex moment-angle manifolds.* Internat. Math. Research Notices, to appear (2021); arXiv:1811.12038.

[3] Roman Krutowski and Taras Panov. *Dolbeault cohomology of complex manifolds with torus action.* Contemp. Math., vol. 772, American Mathematical Society, Providence, RI, 2021, to appear; arXiv:1908.06356.