kpa70

Conference celebrating the 70th birthday of Prof. Krzysztof M. Pawałowski

11–13 January 2021, Online conference via Zoom

`https://kpa70.wmi.amu.edu.pl/`

Invited Speakers:

- William Browder (Princeton University),
- Sylvain Cappell (New York University),
- James F. Davis (Indiana University Bloomington),
- Bogusław Hajduk (University of Warmia and Mazury),
- Jarosław Kędra (University of Aberdeen),
- Mikiya Masuda (Osaka City University),
- Masaharu Morimoto (Okayama University),
- Robert Oliver (Paris University 13),
- Taras Panov (Moscow State University),
- Józef Przytycki (George Washington University and University of Gdańsk),
- Toshio Sumi (Kyushu University).

Organizers:

- Marek Kaluba,
- Wojciech Politarczyk,
- Bartosz Biadasiewicz,
- Łukasz Michalak,
- Piotr Mizerka,
- Agnieszka Stelmaszyk-Śmierzchalska.

Monday, January 11

Time			
Washington	Warsaw	Tokyo	
6:45 – 7:00	12:45 – 13:00	20:45 – 21:00	Opening
7:00 – 7:45	13:00 – 13:45	21:00 – 21:45	**Mikiya Masuda**, Invariants of the cohomology rings of the permutohedral varieties
8:00 – 8:45	14:00 – 14:45	22:00 – 22:45	**Taras Panov**, Holomorphic foliations and complex moment-angle manifolds
9:15 – 10:00	15:15 – 16:00	23:15 – 00:00	**Robert Oliver**, The loop space homology of a small category
10:15 – 11:00	16:15 – 17:00	00:15 – 01:00	**Józef H. Przytycki**, Adventures of Knot Theorist: from Fox 3-colorings to Yang-Baxter homology– 5 years after Poznań talks

Tuesday, January 12

Time			
Washington	Warsaw	Tokyo	
6:20 – 6:45	12:20 – 12:45	20:20 – 20:45	**Piotr Mizerka**, New results on one and two fixed point actions on spheres
7:00 – 7:45	13:00 – 13:45	21:00 – 21:45	**Masaharu Morimoto**, Equivariant Surgery and Dimension Conditions
8:00 – 8:45	14:00 – 14:45	22:00 – 22:45	**Toshio Sumi**, Smith Problem and Laitinen's Conjecture
9:15 – 10:00	15:15 – 16:00	23:15 – 00:00	**Sylvain Cappell**, Fixed points of G-CW-complex with prescribed homotopy type
10:15 – 11:00	16:15 – 17:00	00:15 – 01:00	**James F. Davis**, The Borel Conjecture and aspherical 4-manifolds
11:15 – 11:40	17:15 – 17:40	01:15 – 01:40	**Bartosz Biadasiewicz**, Local normality of infravacuum representations

Wednesday, January 13

Time			
Washington	Warsaw	Tokyo	
6:20 – 6:45	12:20 – 12:45	20:20 – 20:45	**Łukasz P. Michalak**, Framed cobordism and classification of free quotients
7:00 – 7:45	13:00 – 13:45	21:00 – 21:45	**Bogusław Hajduk**, Fixed point sets of symplectic actions on disks
8:00 – 8:45	14:00 – 14:45	22:00 – 22:45	**Jarosław Kędra**, On generation of arithmetic groups by conjugacy classes
9:15 – 9:35	15:15 – 15:35	23:15 – 23:35	**Marek Kaluba**, Small hyperbolic groups with property (T)
9:40 – 10:00	15:40 – 16:00	23:40 – 00:00	**Wojciech Politarczyk**, Equivariant Khovanov homotopy type
10:15 – 11:00	16:15 – 17:00	00:15 – 01:00	**William Browder**, Compact group actions and algebraic topology

Local normality of infravacuum representations

Bartosz Biadasiewicz
Adam Mickiewicz University in Poznań

After a short review of the Haag-Kastler axioms, I will focus on a certain Weyl algebra. I am interested in its infravacuum representation which is inequivalent to the usual Fock representation. I want to show that the equivalence is restored after restriction to a certain subalgebra. I am working on this project with Wojciech Dybalski.

Compact group actions and algebraic topology

William Browder
Princeton University

A new point of view is introduced in the study of free actions on finite dimensional spaces to enable the use of many standard constructions of algebraic topology (such as Postnikov systems) in the study of problems in group actions, such as: What is the maximal rank r of an elementary abelian group that acts freely on a product of k different spheres (the Rank Conjecture: r is less than or equal to k)

Fixed Points of G-CW-complex with Prescribed Homotopy Type

Sylvain Cappell
Courant Institute of Mathematical Sciences, New York University

For a finite group G of not prime power order, Oliver showed that the obstruction for a finite CW-complex F to be the fixed point set of a contractible finite G-CW complex depends upon the Euler characteristic $\chi(F)$. We show that the similar problem for F to be the fixed point set of a finite G-CW complex of some given homotopy type still depends upon the Euler characteristic. For simply-connected homotopy types, this was known using different methods from joint work of Oliver and Petrie. We also give additional information on the obstruction in terms of the whole and the individual connected components of the fixed point set.

This is one of a pair of current works on fixed points of finite transformation groups of the speaker with Shmuel Weinberger of the U. of Chicago and Min Yan of Hong Kong U. of Science and Tech; the other work treats, with different methods and contrasting results, fixed points of semi-free actions in non-simply connected settings.

The Borel Conjecture and aspherical 4-manifolds

James F. Davis
Indiana University Bloomington

A space is aspherical if its universal cover is contractible. Obstruction theory shows that any two aspherical CW complexes with isomorphic fundamental groups are homotopy equivalent. The Borel Conjecture states that any two closed aspherical manifolds with isomorphic fundamental groups are homeomorphic. The Borel Conjecture for manifolds with boundary states that two aspherical compact manifolds with isomorphic fundamental groups and homeomorphic boundaries are in fact homeomorphic.

Topology surgery shows that the Borel Conjecture is valid in dimension 4 for manifolds with good (essentially elementary amenable) fundamental groups. Work in progress with Jonathan Hillman examines the possible fundamental groups and the possible homeomorphism classes of the boundary of compact aspherical 4-manifolds.

Fixed point sets of symplectic actions on disks

Bogusław Hajduk
University of Warmia and Mazury

I will give a characterization of manifolds which are fixed points of symplectic actions of finite groups on disks. The case of circle actions will be also discussed. This is a report on a joint work with Krzysztof.

Small hyperbolic groups with property (T)

Marek Kaluba
Technische Universität Berlin and Adam Mickiewicz University in Poznań

I will give an example of a finitely presented group which is infinite, hyperbolic and has property (T). Hyperbolicity of the group can be proved by observing geometric actions of the group on its coset graphs. The proof of property (T) uses computer assisted computations to estimate so called *angle between subgroups* and relies on a criterion of Ershov–Jaikin–Zapirain. The group itself comes from a generic construction of 5-fold generalized triangle groups and strengthens a result of Lubotzky–Manning–Wilson.

This talk is based on `https://arxiv.org/abs/2011.09276`, with P-E. Caprace, M. Conder and S. Witzel as coauthors.

On generation of arithmetic groups by conjugacy classes

Jarosław Kędra
University of Aberdeen

I will discuss word metrics on groups associated with generating sets invariant under conjugation. It turns out that such a metric on $\mathrm{SL}(n,\mathbb{Z})$ has finite diameter if $n \geq 3$. I will discuss the question of how does the diameter depend on the choice of a generating set. In particular, whether there is a uniform bounds. The answer depends on the number of conjugacy classes comprising the generating set and it was obtained a few years ago by Libman, Martin and myself.

One expects that such results should generalise to other arithmetic Chevalley groups and indeed they do which is a recent theorem of Alexander Trost. His proof (different from ours) is based on model theoretic ideas of Dave Witte Morris. I will present the main ideas of his argument and some consequences.

Invariants of the cohomology rings of the permutohedral varieties

Mikiya Masuda
Osaka City University Advanced Mathematical Institute

The permutohedron P of dimension $n-1$ has a natural action of the symmetric group $\mathfrak{S}_n$, so the toric variety X associated to P, called the permutohedral variety, has the induced action of $\mathfrak{S}_n$. In this talk I will report that the ring of invariants $H^*(X;\mathbb{Q})^{\mathfrak{S}_\lambda}$ under the restricted action of a Young subgroup $\mathfrak{S}_\lambda$ is isomorphic to the cohomology ring of the toric orbifold associated to the "quotient" polytope $P/\mathfrak{S}_\lambda$. This talk is based on joint work with T. Horiguchi, J. Shareshian and J. Song.

Framed cobordism and classification of free quotients

Łukasz P. Michalak
Adam Mickiewicz University in Poznań

In this talk we will show how framed cobordism of systems of non-separating 2-sided submanifolds in a closed manifold can be used to classify epimorphisms onto free groups up to equivalence and strong equivalence. Such a classification is known for surface groups and was done by Grigorchuk–Kurchanov–Zieschang by using other methods. We use an extended Pontryagin–Thom construction to associate for any system of submanifolds an induced homomorphism to a free group. We will present geometric operations on submanifolds which realize elementary Nielsen transformations on induced homomorphisms. These results are motivated by the notion of Reeb graph of a function on a manifold, which leads to both free quotient of fundamental group of manifold and system of submanifold.

The talk is based on joint work with Wacław Marzantowicz.

New results on one and two fixed point actions on spheres

Piotr Mizerka
Adam Mickiewicz University in Poznań

It is known that a finite group G admits a smooth one fixed point action on a sphere if and only if G is an Oliver group. One can ask therefore a more specific question: what are the dimensions of spheres on which a given Oliver group can act smoothly with exactly one fixed point? We provide a list of specific Oliver groups and dimensions of spheres for which it was possible to exclude such actions. This part of the talk if based on the joint work with A. Borowiecka, as well as its further generalizations contained in my PhD thesis.

In the second part of the talk we focus on the Laitinen Conjecture which predicts negative answers to the Smith Question (Smith asked whether, for a finite group G acting smoothly on a sphere S with exactly two fixed points x and y, the tangent spaces T_xS and T_yS have to be isomorphic as $\mathbb{R}G$-modules). We define an infinite family of finite groups for which the Laitinen Conjecture remained unsettled and show that this conjecture holds for this family of groups. Moreover, in the case of the groups from the family, the Laitinen Conjecture implies the existence of their actions on homotopy spheres with exactly two fixed points and non-isomorphic group module structures of the tangent spaces at the two fixed points.

Equivariant Surgery and Dimension Conditions

Masaharu Morimoto
Okayama University

Many researchers (e.g. J. Milnor, M. Kervaire, S. Novikov, W. Browder, C. Wall, W.-C. Hsiang, W. Sharpe, Cappell-Shaneson, D. Anderson, A. Bak, F. Connolly, S. Ferry, I. Hambleton, I. Madsen, E. Pedersen, C. Thomas, and etc.) had contributed to development of the ordinary surgery theory on compact smooth manifolds X of dimension ≥ 5 and to its applications. We can perform surgeries on the manifold X up to the middle dimension when a degree-one map $f : X \to Y$ and a bundle isomorphism $b : T(X) \oplus \varepsilon_X(\mathbb{R}^s) \to f^*\xi \oplus \varepsilon_X(\mathbb{R}^s)$, where $\xi \downarrow Y$, are given.

Let G be a finite group and X a compact connected smooth G-manifold with a G-map $f : X \to Y$ and a G-bundle isomorphism $b : T(X) \oplus \varepsilon_X(\mathbb{R}^s) \to f^*\xi \oplus \varepsilon_X(\mathbb{R}^s)$. To perform G-surgeries on X of isotropy type (H), some dimension conditions on the fixed point sets by subgroups of G must be invoked. First we require the hypothesis $\dim X^H_\alpha \geq 5$ on the relevant connected component X^H_α of X^H. T. Petrie's theory of G-surgery of isotropy type (H) works well if we invoke the strong gap condition:

$$\text{(SGC)} \quad \begin{array}{l} (1)\ 2\dim X^K_\beta + 2 < \dim X^H_\alpha \quad (H < \forall K \leq N_G(H), \forall X^K_\beta \subset X^K_\alpha), \text{ and} \\ (2)\ 2\dim X^K_\beta < \dim X^H_\alpha \quad (H < \forall K \leq G \text{ with } K \not\leq N_G(H), \forall X^K_\beta \subset X^K_\alpha). \end{array}$$

Petrie–K. Dovermann studied the induction–restriction theory of the G-surgery obstructions. We found the method killing the G-surgery obstructions by G-connected sums associated with certain idempotents of the Burnside ring of G. Morimoto–K. Pawałowski applied the method to obtain a

theorem to delete (or insert) closed manifolds as connected components of G-fixed point sets from (or into) spheres for gap Oliver groups G. The result presented a passage to finding non-isomorphic Smith equivalent representations.

Generalizing Wall's surgery obstruction group by the notion 'form parameter of quadratic forms' appearing in Bak's book, we improved G-surgery theory under the gap condition:

$$\text{(GC)} \qquad 2\dim X_\beta^K < \dim X_\alpha^H \quad (H < \forall K \leq G,\, \forall X_\beta^K \subset X_\alpha^K).$$

We spent a decade to improve it to G-surgery theory under the weak gap condition:

$$\text{(WGC)} \quad \begin{array}{ll} (1)\ 2\dim X_\beta^K \leq \dim X_\alpha^H & (H < \forall K \leq N_G(H),\, \forall X_\beta^K \subset X_\alpha^K), \text{ and} \\ (2)\ 2\dim X_\beta^K < \dim X_\alpha^H & (H < \forall K \leq G \text{ with } K \not\leq N_G(H),\, \forall X_\beta^K \subset X_\alpha^K). \end{array}$$

To obtain the theory, we need the notion 'positioning data' and the notion 'generalized form module with positioning data'. We should remark that Dovermann had already studied a C_2-surgery theory under the weak gap condition, where C_2 is the group of order 2.

In this talk, we revisit the G-surgery theories and review several applications.

The loop space homology of a small category

Bob Oliver
Université Paris 13

In an article published in 2009, Dave Benson described, for a finite group G, the mod p homology of the space $\Omega(BG^\wedge_p)$ — the loop space of the p-completion of BG — in purely algebraic terms. In joint work with Carles Broto and Ran Levi, we have tried to better understand Benson's result by generalizing it. Among other things, we showed that when $\mathcal{C}$ is a small category, $|\mathcal{C}|$ is its geometric realization, R is a commutative ring, and $|\mathcal{C}|^+_R$ is a plus construction of $|\mathcal{C}|$ with respect to homology with coefficients in R, then $H_*(\Omega(|\mathcal{C}|^+_R); R)$ is the homology any chain complex of projective $R\mathcal{C}$-modules that satisfies certain conditions. Benson's theorem is then the special case where $\mathcal{C}$ is the category associated to a finite group G and $R = \mathbb{F}_p$, and thus p-completion appears as a special case of the plus construction.

Holomorphic foliations on complex moment-angle manifolds

Taras Panov[1]
Department of Mathematics and Mechanics, Moscow State University

Moment-angle manifolds provide a wide class of examples of non-Kaehler compact complex manifolds. A complex moment-angle manifold Z is constructed via a certain combinatorial data, called a complete simplicial fan. In the case of rational fans, the manifold Z is the total space of a holomorphic bundle over a toric variety with fibres compact complex tori. In this case, the invariants of the complex structure of Z, such Dolbeault cohomology and the Hodge numbers, can be analysed using the Borel spectral sequence of the holomorphic bundle.

In general, a complex moment-angle manifold Z is equipped with a canonical holomorphic foliation $\mathcal{F}$ which is equivariant with respect to the algebraic torus action. Examples of moment-angle manifolds include the Hopf manifolds, Calabi-Eckmann manifolds, and their deformations. The holomorphic foliated manifold $(Z, \mathcal{F})$ has been also studied as a model for non-commutative toric varieties in the works several authors (arXiv:1308.2774, arXiv:1705.11110).

We construct transversely Kaehler metrics on moment-angle manifolds Z, under some restriction on the combinatorial data. We prove that all Kaehler submanifolds in such a moment-angle manifold lie in a compact complex torus contained in a fibre of the foliation $\mathcal{F}$. For a generic moment-angle manifold Z in its combinatorial class, we prove that all its subvarieties are moment-angle manifolds of smaller dimension. This implies, in particular, that Z does not have non-constant meromorphic functions, i. e. its algebraic dimension is zero.

Battaglia and Zaffran (arXiv:1108.1637) computed the basic Betti numbers for the canonical holomorphic foliation on a moment-angle manifold Z corresponding to a shellable fan. They conjectured that the basic cohomology ring in the case of any complete simplicial fan has a description similar to the cohomology ring of a complete simplicial toric variety due to Danilov and Jurkiewicz. We prove the conjecture. The proof uses an Eilenberg–Moore spectral sequence argument; the key ingredient is the formality of the Cartan model for the torus action on Z.

The talk is based on joint works with Hiroaki Ishida, Roman Krutowski, Yuri Ustinovsky and Misha Verbitsky.

References

[1] Taras Panov, Yuri Ustinovsky and Misha Verbitsky. *Complex geometry of moment-angle manifolds.* Math. Zeitschrift 284 (2016), no. 1, 309–333.

[2] Hiroaki Ishida, Roman Krutowski and Taras Panov. *Basic cohomology of canonical holomorphic foliations on complex moment-angle manifolds.* Internat. Math. Research Notices, to appear (2021); arXiv:1811.12038.

[3] Roman Krutowski and Taras Panov. *Dolbeault cohomology of complex manifolds with torus action.* Contemp. Math., vol. 772, American Mathematical Society, Providence, RI, 2021, to appear; arXiv:1908.06356.

[1]The research was carried out at the Steklov Institute of Mathematics and supported by the Russian Science Foundation grant no. 20-11-19998.

Equivariant Khovanov homotopy type

Wojciech Politarczyk
Warsaw University

The Khovanov homotopy type is a space level refinement of Khovanov homology introduced by Lipshitz and Sarkar. The main goal of the talk will be to show that the Khovanov homotopy type can be effectively used to study periodic links, i.e., links which are invariant under a finite order rotation of the three-sphere. In particular, by applying the classical localization theorem for Borel cohomology and its extension we obtain interesting relations between Khovanov homology of a periodic link and Khovanov homology of the quotient.

Adventures of Knot Theorist: from Fox 3-colorings to Yang-Baxter homology– 5 years after Poznań talks

Józef H. Przytycki
George Washington University and University of Gdańsk

Five years ago I was invited by Krzysztof to give a series of talk on Knot Theory at the University of Poznań. I gave 10 double talks starting from:
Lecture 1: Short historical introduction,
Lecture 2: 3-coloring of Fox,
and ending with:
Lectures 9 and 10: Distributive homology and homology of Yang-Baxter operators.

My lectures were an introduction to ambitious goal which can be summarize as follows:

We propose to develop connections between Yang–Baxter homology, which is a generalization of the distributive homology, including the rack or quandle homology, and Khovanov homology. We envision the connection via the cocycle invariants of links obtained from quandle or biquandle homology, motivating it via Knot Theory.

For a non-specialist we would describe the project as follows: Mathematics that we study in school is usually associative and up till now most of the modern mathematics assumed associativity.

Khovanov homology has an associative and co-associative underlying algebraic structure. Khovanov homology was constructed for and motivated by theory of knots and links.

Another algebraic structures discovered recently, quandles and racks, are distributive but not associative. Just like Khovanov homology, quandles and their homology were motivated by knot theory. No direct link between Khovanov homology and quandle homology is currently known. We propose a program which may link these two modern concepts. The tools we envision are Yang-Baxter operators, powerful tools used in statistical physics, related to at least three Nobel prizes (Yang 1957, Bethe 1967, Onsager 1968).

Smith problem and Laitinen's conjecture

Toshio Sumi
Division for Theoretical Natural Science, Faculty of Arts and Science, Kyushu University

Professor Krzysztof Pawałowski is studying the Smith problem. Corresponding to the Smith problem, Laitinen conjectured that if a finite Oliver group G has the property that the number of real conjugacy classes of elements not of prime power order is greater than or equal to 2, there exist nontrivial Laitinen–Smith equivalent G-modules. Here, two G-modules are called Laitinen–Smith equivalent if they are tangential representations of a sphere with a smooth G-action having exactly two fixed points under mild connectivity condition. First, Morimoto pointed out that $\mathrm{Aut}(A_6)$ is a counterexample for the Laitinen's conjecture. After that, Pawałowski and I concluded that it is a unique counterexample among unsolvable groups. Although there exist a few counterexamples among solvable Oliver groups, many groups satisfy the Laitinen's conjecture. I will talk about a sufficient condition for a group that the Laitinen's conjecture is true. This is a joint work with professor Krzysztof Pawałowski.